APPENDIX A

```
% BEGINNING OF PSEUDO CODE

% compute scale factor A, and time constants a, b from physical system
% parameters

A = Vmax * Kt / (Re * Rm + Kt * Kb) * 1 * k;

p1 =  1/Jm/Ie * (-Ie * Rm - Re * Jm + sqrt(Ie^2 * Rm^2 - 2 * Re * Rm * Ie * Jm
      + Re^2 * Jm^2 - 4 * Kt * Kb * Ie * Jm)) / 2;
p2 =  1/Jm/Ie * (-Ie * Rm - Re * Jm - sqrt(Ie^2 * Rm^2 - 2 * Re * Rm * Ie * Jm
      +  Re^2 * Jm^2 - 4 * Kt * Kb * Ie * Jm)) / 2;

a = max(-p1,-p2)
b = min(-p1,-p2)

% make initial guesses for step durations

et1 = 1;
et2 = .005;
et3 = 1;

% set maximum iteration count

Nmax = 1000;

for j = 1:Nmax
% save old values of step time intervals
et3old = et3;
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et2old = et2;
et1old = et1;

% iterate for switch times using fixed voltage level Vmax

et3 =   -log(1.0 / 2.0 - exp(-et1 * a) / 2 + exp(-et2 * a)) / a;
et2 =   1/b * log(2.0) + 3 * et3 - 1/b * log(2 * exp(1/A * b * X) * exp(et3
        * b) - sqrt( 4.0) * sqrt(exp(1/A * b * X)) * exp(et3 * b) *
        sqrt(exp(1/A * b * X)+exp(et3 * b)^2 - 2 * exp(et3 * b)));
et1 = - (-2 * A * et2 + 2 * A * et3 - X) / A;

if norm([et3old - et3 et2old - et2 et1old - et1], inf) <= eps * 2
        break
end
if j==Nmax
                error(['error - failure to converge after ', num2str(Nmax),'
        iterations'])
end
end

% round up pulse duration to nearest sample interval,
% convert to intervals between steps to make sure that voltage
% requirements will not increase (beyond Vmax).

dt1=ceil((et1 - et2) / dt) * dt;
dt2=ceil((et2 - et3) / dt) * dt;
dt3=ceil((et3) / dt) * dt;

et123 = [et1, et2, et3]
% convert back to total step duration.
```

et1 = dt1 + dt2 + dt3;

et2 = dt2 + dt3;

et3 = dt3;

% In the following, the original constraints equations involving XF1, XF2,

% and XF3 have been modified to include a variable voltage level applied at

% each step (instead of the fixed maximum (+/-) Vmax).

% The original equations for XF1, XF2, and XF3 follow:

% $XF_1(t_{end}) = V_0F_1(t_{tend} - t_0) - 2V_0F_1(t_{end} - t_1) + 2V_0F_1(t_{end} - t_2)$

% $XF_2(t_{end}) = V_0F_2(t_{tend} - t_0) - 2V_0F_2(t_{end} - t_1) + 2V_0F_1(t_{end} - t_2)$

% $XF_3(t_{end}) = V_0F_3(t_{tend} - t_0) - 2V_0F_2(t_{end} - t_1) + 2V_0F_1(t_{end} - t_2)$

% And the modified equation including adjustable relative levels of voltage

% L1, L2 and L3 are:

% $XF_1(t_{end}) = L_1V_0F_1(t_{tend} - t_0) - L_2V_0F_1(t_{end} - t_1) + L_3V_0F_1(t_{end} - t_2)$

% $XF_2(t_{end}) = L_1V_0F_2(t_{tend} - t_0) - L_2V_0F_2(t_{end} - t_1) + L_3V_0F_1(t_{end} - t_2)$

% $XF_3(t_{end}) = L_1V_0F_3(t_{tend} - t_0) - L_2V_0F_2(t_{end} - t_1) + L_3V_0F_1(t_{end} - t_2)$

% And the corresponding constraint equations are:

% $XF_1(t_{end}) = \text{Finalpos}$

% $XF_2(t_{end}) = 0$

% $XF_3(t_{end}) = 0$

% Where all of the times indicated have discrete values, e.g. corresponding to

% the controller update rate.

```
% It should be noted that after the digital switch times are fixed, the constraint
% equations derived from the equations above form a linear set of equations in
% the unknown relative voltage levels L1, L2 and L3 and any standard linear
% method can be used to solve for the relative voltage levels.  In the equations
% for (L1, L2 and L3) that follow, the solution was obtained by algebraic
% means (and are not particularly compact.)

% compute new relative voltage step levels
% L1, L2 and L3 are nominally assigned to "1", "-2" and "+2", respectively
s1 =  X * (exp(-et3 * b) * exp(-et2 * a) + exp(-et3 * a) + exp(-et2 *b ) -exp(-et2
      *b) * exp(-et3 * a) - exp(-et2 * a) - exp(-et3 *b ));
s2 =  1 / (et2 * exp(-et1 * b) * exp(-et3 * a) + exp(-et2 * b) * et3 *
          exp(-et1 * a)- et2 * exp(-et3 * a) - et2 * exp(-et1 * b)  -et3 *
          exp(-et1 * a) - exp(-et2 * b) * et3+exp(-et3 * b) * et1 * exp(-et2 *
      a) + exp(-et3 * a) * et1 + exp(-et2 * b) * et1 - exp(-et2 * b) * et1 *
          exp(-et3 * a) - et3 * exp(-et1 * b) * exp(-et2 * a) - exp(-et2 * a) *
      et1 - exp(-et3 * b) * et1 - exp(-et3 * b) * et2 * exp(-et1 * a) + et3 *
          exp(-et1 * b) + et2 * exp(-et1 * a) + exp(-et3 * b) * et2 + et3 *
          exp(-et2 * a)) / A;

      L1 =   s1 * s2;

s1 =  1 / (et2 * exp(-et1 * b) * exp(-et3 * a) + exp(-et2 * b) * et3 *
          exp(-et1 * a) - et2 * exp(-et3 * a) - et2 * exp(-et1 * b) - et3 *
          exp(-et1 * a) - exp(-et2 * b) * et3 + exp(-et3 * b) * et1 *
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exp(-et2 * a) + exp(-et3 * a) * et1+exp(-et2 * b) * et1 -
        exp(-et2 * b) * et1 * exp(-et3 * a) - et3 * exp(-et1 * b) *
        exp(-et2 * a) - exp(-et2 * a) * et1 - exp(-et3 * b) * et1 - exp(-et3 *
    b) * et2 * exp(-et1 * a) + et3 * exp(-et1 * b) + et2 * exp(-et1 * a) +
        exp(-et3 *b ) * et2 + et3 * exp(-et2 * a)) * X;
s2 = (exp(-et2 * b) * exp(-et1 * a) - exp(-et1 * a) - exp(-et2 * b) -
        exp(-et1 * b) * exp(-et2 * a) + exp(-et1 * b) + exp(-et2 * a)) / A;
L3 = s1*s2;

s1 = exp(-et1 * a) - exp(-et3 * a) + exp(-et3 * b) - exp(-et1 * b) -
        exp(-et3 * b) * exp(-et1 * a) + exp(-et1 * b) * exp(-et3 * a);
s2 = X / (et2 * exp(-et1 * b) * exp(-et3 * a) + exp(-et2 * b) * et3 *
        exp(-et1 * a) - et2 * exp(-et3 * a) - et2 * exp(-et1 * b) - et3 *
        exp(-et1 * a) - exp(-et2 * b) * et3 + exp(-et3 * b) * et1 * exp(-et2 *
    a) + exp(-et3 * a) * et1 + exp(-et2 * b) * et1 - exp(-et2 * b) * et1 * exp(-
    et3 * a) - et3 * exp(-et1 * b) * exp(-et2 *a ) - exp(-et2 * a) * et1-exp(-et3 *
    b) * et1 - exp(-et3 * b) * et2 * exp(-et1 * a) + et3 *
        exp(-et1 * b) + et2 * exp(-et1 * a) + exp(-et3 * b) * et2 + et3 *
        exp(-et2 * a)) / A;

L2 =  s1 * s2;

% convert accumulated voltage steps to sequential voltage level
V1 = Vmax * (L1);
V2 = Vmax * (L1 + L2);
V3 = Vmax * (L1 + L2 + L3);

% END OF PSEUDO CODE
```

APPENDIX B

```
AREA .. SUM(I,A(I)) =E= 0;
VELOCITY(VINDX) .. VEL(VINDX) =E= VSCALE *
SUM(I$(ORD(I) LE ORD(VINDX)), A(I));
POSITION .. SUM(I,VEL(I)) =E= FINALPOS * SCALEFACT;
VLIMITP(I) .. SUM(VINDX$(ORD(VINDX) LE ORD(I)),A(I-
(ORD(VINDX)+1))*(VOLTS(VINDX)+KBACK*VSCALE))
=L= VOLTLIM;
VLIMITN(I) .. SUM(VINDX$(ORD(VINDX) LE ORD(I)), A(I-
(ORD(VINDX)+1))*(VOLTS(VINDX)+KBACK*VSCALE))
=G= -VOLTLIM
```

% A(I) are the current commands at time T(I) spaced equally at time DT.

% VOLTS(VINDX) is a table of voltages representing the unit pulse response to

% a unit output in current command. VOLTLIM is the voltage limit at saturation.

APPENDIX C

```
GOALPOS .. SUM(I,A(I)*MODELAA*DT) =E=FINALPOS;
MODE1(ILAST) .. SUM(I,-A(I)*MODELAA*MODELb/(MODELb-
MODELa)*(EXP(-MODELa*(T(ILAST)+DT-T(I)))
-EXP(-MODELa*(T(ILAST)-T(I))))) =E= 0.0;
MODE2(ILAST) .. SUM(I,A(I)*MODELAA*MODELa/(MODELb-
MODELa)*(EXP(-MODELb*(T(ILAST)+DT-T(I)))
-EXP(-MODELb*(T(ILAST)-T(I))))) =E= 0.0;
DERIV1(J) .. 1000.0*SUM(I,A(I)*T(I)*EXP(ZETA(J)*W(J)*T(I))*
SIN(WD(J)*T(I))) =E= 0.0 ;
DERIV2(J) .. 1000.0*SUM(I,A(I)*T(I)*EXP(ZETA(J)*W(J)*T(I))*
COS(WD(J)*T(I))) =E= 0.0 ;
```

% MODELAA is the mechanical gain of the system, MODELb, and MODELa
% are the two time constants of the system in radians. One time constant is
% associated with the L/R rise time of the motor inductance and the other is
% the mechanical time constant of the rigid system. The A(I) are the voltages %
which need to be determined. The T(I) are the times for each of the A(I).
% DT is the time spacing of the outputs. W(J) are the undamped flexible
% modes, WD(J) are the damped flexible modes (in radians/s).